***Constituents of atomic nucleus***

All the atomic nuclei are made up of elementary particles called protons and neutrons. A proton has a positive charge of the same magnitude as that of an electron and mass of proton is. A neutron is electrically neutral and mass of neutron is same as that of proton.

An atomic nucleus is represented by.

***Atomic number:*** The number of protons in a nucleus is called the atomic number. It is denoted by *Z*. The identity of an element depends on the atomic number in a nucleus.

***Mass number:*** The total number of protons and neutrons in a nucleus is called mass number. It is denoted by *A*.

We can write

Where, *Z* = Number of protons

*N* = Number of neutrons.

***Nucleon:*** The combined (or common) name of proton and neutron which construct nucleus is called nucleon.

***Nuclide:*** The similar nuclei which are specified by the mass number and atomic number are known as nuclide. For example, *different isotopes of the same elements are different nuclide.*

***Classification of nucleus***

***Isotopes:*** Nuclei having same number of protons but different number of neutrons are called isotopes.

Examples:are isotopes of Hydrogen.

are isotopes of Carbon.

***Isobars:*** Nuclei having same mass number but different atomic number/proton number are called isobars.

Examples:are isobars.

are isobars.

***Isotones:*** Nuclei having same number of neutrons but different atomic number/proton number are called isobars.

Examples:are isotones.

are isotones.

***Mirror nucleus:*** Nuclei having the same mass number but the proton and neutron number are interchanged are called mirror nucleus.

Examples:are mirror nucleus.

are mirror nucleus.

***Measurement of nuclear mass***

In various phenomenona of nucleus it is seen that mass is converted into energy and vice-versa. So, it is convenient to express mass and energy in same unit.

If *m* kg is the mass of the particle then the equivalent energy of the mass is *mc2* Joule.

We know

One electron volt unit is defined as the amount of work an electron does to cross a potential difference of 1 volt.

Therefore

Therefore the equivalent energy of *m* kg is

We know, atomic mass unit

We know in 1 gm-atom the number of atoms is . This is known as Avogadro’s number. So, in 12 gm Carbon there are atoms of Carbon.

So, the mass of a Carbon-12 atom

Thus, atomic mass unit

***Radius of the nucleus:*** The size of the nucleus depends upon the number of protons and the number of neutrons inside it. Thus the volume of a nucleus is directly proportional to the number of nucleons contained in it, which is its mass number *A*.

If *R* is the nuclear radius, then

Where is a constant and .

Nuclei are so small that the unit of length appropriate in describing them is the femtometer (*fm*) equal to . The femtometer often called *Fermi*.

Thus the radius of the nucleus is

***Mass defect:*** Neutrons and protons together create a nucleus. So, the mass of a nucleus should be equal to the mass of the protons and neutrons added together. But actually the mass of a nucleus is little less than the mass of the protons and neutrons added together. The difference in mass is known as mass defect. It is denoted by *M*.

Mathematically,

Where, *Z* = Number of protons

*(A-Z)* = Number of neutrons.

*Mp* = Mass of protons

*Mn* = Mass of neutrons

*M* = Mass of nucleus.

***Binding energy:*** The energy equivalent to mass defect is called the binding energy of the nucleus. If we want to break the nucleus into separate protons and neutrons, again we have to introduce the same amount of energy that is lost during creation. *The minimum amount of energy needed to break a nucleus into its constituents neutrons and protons, is called the binding energy of the nucleus.*

The relation between the binding energy and mass defect is given by

If all masses are in atomic mass unit (a.m.u.) then

The binding energy is always a positive quantity.

***Nuclear reaction***

When a nucleus gets in close contact with another nucleus, the incident particle and the target nucleus form a composite system which is an excited state and after a short while a reaction is produced in which the incident particle itself or some other particle or gamma ray is emitted with excess energy and a resulting nucleus is obtained. This phenomenon is called a nuclear reaction.

A nuclear reaction can be written as

Where, *a* is the incident particle

*X* is the target nucleus

*Y* is the residual product nucleus

*b* is the resulting or outgoing particle.

*a* and *b* are incident and outgoing particle respectively and these may be any one of the following: Proton, Neutron, Deuteron, Alpha particle, Gamma ray or a few nucleons.

***Examples:***

|  |  |
| --- | --- |
| ***Nuclear reaction*** | ***Short form of Nuclear Reaction*** |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

***Nuclear fission and fusion reaction***

***Nuclear fission:*** If neutrons or gamma rays of definite speed strike a heavy nucleus (A>230) and breaks the nucleus into two main parts, then a vast nuclear energy is released. This kind of breakup of nucleus is called nuclear fission.

Let us consider the capture of neutron by a heavy nucleus. The reaction can be represented as

***Nuclear fusion:*** More than one light nuclei are fused (or combined) together to form a single heavy nucleus and produce enormous nuclear energy. This process is called nuclear fusion.

The reaction can be represented as

***Differences between nuclear fission and fusion reaction:*** The differences between nuclear fission and fusion reaction are given below:

|  |  |
| --- | --- |
| ***Nuclear fission*** | ***Nuclear fusion*** |
| In this process heavy nucleus is splitted into lighter nucleus. | In this process lighter nuclei are combined to produce heavy nucleus. |
| Energy released is large. | Energy released is small. |
| nuclei are used. | nuclei are used. |
| In this process radioisotopes are produced. | In this process radioisotopes are not produced. |
| The link of the fission process is with neutrons. | The link of the fusion process is with protons. |

***Bohr atomic model***

In 1913, Bohr suggested that, *positively charged nucleus is at the centre and negatively charged electrons revolve round the nucleus in various circular orbit*. The electrons revolving round the nucleus only in certain permitted orbits are called *energy levels*. Each energy level has a certain *fixed amount of energy*. The larger the orbit (i.e. larger radius), the greater is the energy of electrons. Electrons gives out energy in the form of electromagnetic radiation when it jumps from *higher to lower level*.

***Postulates of Bohr atomic model***

Bohr atomic model is based on the following postulates:

1. An atom consists of positively charged nucleus at the centre.
2. The negatively charged electrons move round the nucleus in various orbits known as stationary energy levels. The electrons can’t emit radiations when moving in their own stationary level
3. The Coulombian and Newtonian forces are applicable in the domain of the atom.
4. The electrons revolve round the nucleus in various circular orbits and the angular momentum

where *n* (= 1, 2, 3, 4,….. ) is called the quantum number, and h is the Planck’s constant.

1. When an electron jumps from a higher energy level to a lower energy level, it gives out electromagnetic radiations of a particular frequency

where is the energy of the upper level and is the energy of the lower level and is the frequency of the electromagnetic radiation.

***Radius of and total energy of atom on the basis of Bohr atomic model***

Let us consider the motion of an electron of mass ‘*m’* and charge ‘*e’* moving uniformly with speed ‘*v’*, in a circular orbit of radius *‘r’* around a nucleus of charge *Ze*. Then it experiences an acceleration *v2/r* towards the centre of the circle.

The electrostatic force of attraction acts between the nucleus and the revolving electron and tend to attract the electron towards the nucleus. According to Coulomb’s law this force is given by

The centripetal force is holding the electron in the orbit from the nucleus is provided by the electrostatic force between them.

Bohr’s postulates states that, the electrostatic force of attraction provides necessary centripetal force for revolution of electron in the orbit, *i.e.*

Again according to the Bohr postulate, the orbital angular momentum of the revolving electron must be equal to the integer multiple of *i.e.,*

where *n* (= 1, 2, 3, 4,….. )

Equating equations (2) and (3) we get

Where is the radius of the nth orbital electron.

For Hydrogen atom *Z =1.* Thus, the radius of the nth orbit of Hydrogen atom is

Now substituting the value of *r* from equation (4) in equation (2) we get

Where is the velocity of the nth orbital electron.

The kinetic energy of the revolving electron is

and the potential energy of the electron is

The total energy *E* of the revolving electron in a stable orbit is given by

Now substituting the value of *r* from equation (4) in equation (9) we get

Where is the energy of the nth orbital electron.

For Hydrogen atom *Z =1.* Thus, the energy of the nth orbit of Hydrogen atom is

For *n =1,* we can write equation (11) as

From equation (11) and (12) we can write

When an electron jumps from upper energy level *n2* to lower energy level *n1*, we can write

From equation (10) we can write

From equation (13), (14) and (15) we can write

Where, is the energy of the emitted photon.

For Hydrogen atom *Z =1.* Thus, the energy of the Hydrogen atom in this case is

***Mathematical Problems***

***Problem-1:*** *Calculate the radius of the first Bohr orbit for H and He atoms.*

***Solution:*** We know,

Here,

For Hydrogen atom *Z =1*

For Helium atom *Z =2*

***Problem-2:*** *Calculate the radius of the third orbit of hydrogen atom. Also calculate the energy of electrons in that orbit. (Given c)*

***Solution:*** We know, for Hydrogen atom

Here,

Energy of an atom is

For Hydrogen atom *Z =1*

***Problem-3:*** *How much energy is required to remove an electron from n=3 or n=2 state in Hydrogen atom.*

***Solution:*** We know, Energy of an atom is

Here,

For Hydrogen atom *Z =1*

***Photo-electric effect***

*When light of sufficiently high frequency falls upon the metal surface, electrons are emitted from it. This phenomenona is known as photoelectric effect. The emitted electrons are called photo-electrons and the current constitutes by their electrons called photo-current.*

*OR: Ejection of electrons from a metal plate when illuminated by light or any other radiation of suitable wavelength (or frequency) is called photo-electric effect.*

******

**Fig. 1**. Photo-electric effect

***Laws/ Characteristics of photo-electric emission***

The laws or characteristics of photoelectric emission are:

1. For a given metallic surface, there is a minimum frequency for which the incident light can eject the photo-electrons out of the metal. Light of frequency smaller than the particular value cannot eject electrons, no matter how long it falls on the surface.
2. The number of photo-electrons ejected depends upon the intensity of the incident light. Thus, the photo-electric current depends upon the intensity of the incident light.
3. Light of frequency higher than the *critical frequency* ejects electrons of different velocities. The maximum velocity, with which an electron is ejected, depends upon the frequency of the incident light. The *threshold frequency* is the minimum frequency which can eject and electron out of the metal.
4. The maximum kinetic energy of the ejected electrons increases linearly with the frequency of the incident light.

***Einstein’s photoelectric equation***

Einstein explained the photoelectric effect based on Planck’s theory of light. According to quantum theory, radiation is regarded as quanta of photons each of energy

moving in space with the velocity of light. Where, *h* = Planck’s constant & *ν* = frequency of radiation.

When a photon of energy *hν* is incident on a certain metallic surface, it is completely absorbed and imparts its energy to a single electron. Thus the photon energy is utilized in two purposes:

1. Partly for getting the electron free from the atom and away from the metal surface.
2. The balance of the photon energy is used up in giving the electron a kinetic energy.

The above phenomenona can be written as

Where, is the maximum photoelectron energy and is the work function which is the minimum energy needed for an electron to leave the metal.

Again *w0 = hν0*, Where, *ν0* is the threshold frequency which is the minimum frequency of the photon which can eject an electron out of the metal.

Equation (1) becomes

This is Einstein’s photoelectric equation.

***Stopping potential***

*The potential which is sufficient to emit the most energetic photo-electrons is called stopping potential.*

***Equation of stopping potential***

Einstein’s photoelectric equation is

We can write

From equation (1) and (2) we can write

This is the equation of stopping potential in photoelectric phenomenon.

***Mathematical Problems***

***Problem-1:*** *A photoelectric surface has a work function of 4eV. What is the maximum velocity of Photoelectrons emitted by light of frequency 1015 Hz incident on the surface?*

***Solution:*** We know,

Here,

***Problem-2:*** *Find the maximum velocity of photo-electrons emitted by radiation of frequency Hz from a photoelectric surface having a work function 4.0 eV.*

***Solution:*** Do yourself. (**Ans:**)

***Problem-3:*** *The wavelength of light falling on the surface of a metal of work function 2.3 eV is 4300 A0 with what velocity the electron will be emitted?*

***Solution:*** We know,

Here,

***Problem-4:*** *Calculate the threshold frequency and corresponding wavelength of radiation incident on a certain metal whose work function is. Given Plank’s constant.*

***Solution:*** We know,

Here,

Again, we know,

***Problem-5:*** *The stopping potential for electrons emitted from a metal due to photoelectric effect is found to be 1V for a light of 2500 A0. Calculate the work function of the metal in eV.*

***Solution:*** *W*e know,

Here,

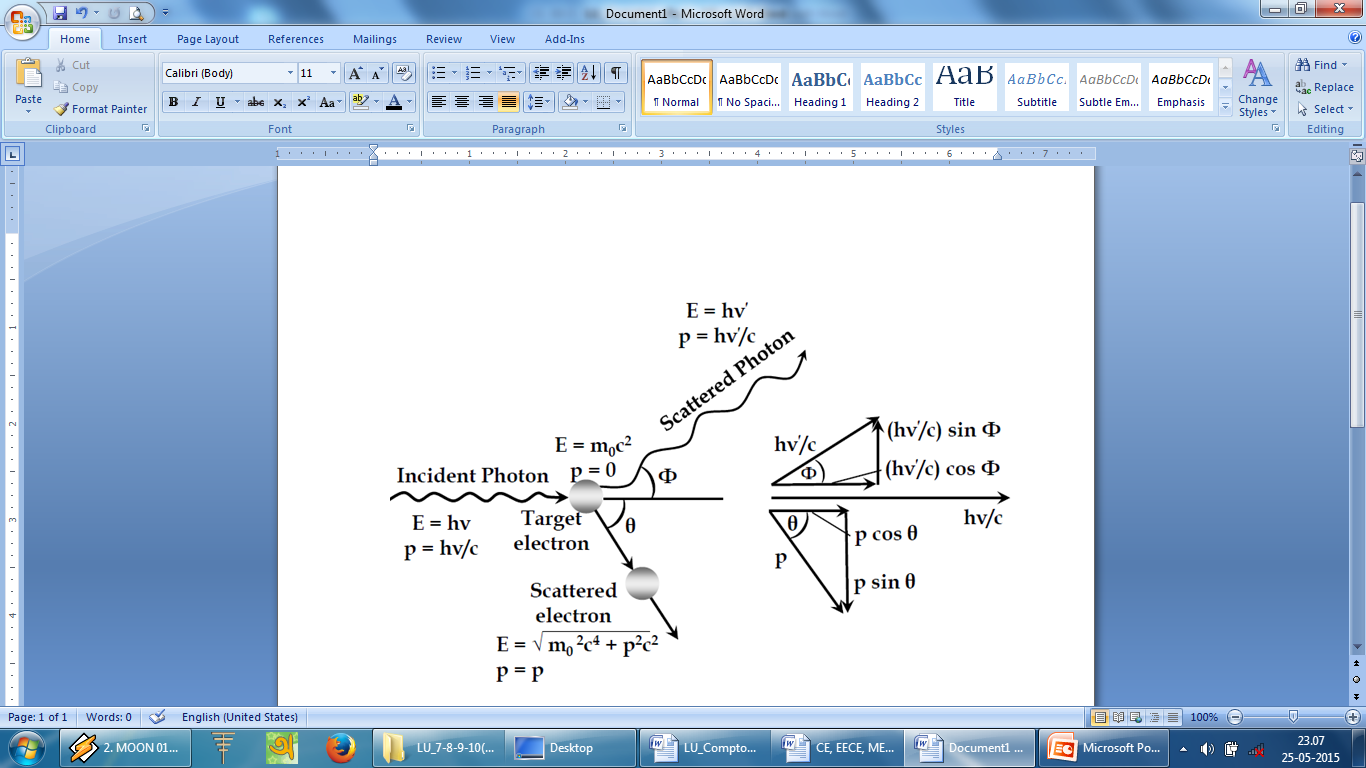
Again, we know,

***Compton Effect***

When an energetic photon strikes an electron (*assumed to be initially at rest*), it is scattered away from its original direction of motion while the electron receives an impulse and begins to move. In the collision, the photon may be regarded as having lost an amount of energy that is the same as the kinetic energy gained by the electron. Consequently, *the scattered photon has less energy than the incident photon. Therefore, the frequency of the scattered photon is shorter than that of incident photon. This alteration in frequency or wavelength of the photon due to the collision with electron is called Compton Effect.*

***Compton Equation***

Let an energetic (X-ray) photon of energy and momentum *hν/c* strikes a free electron at rest. The energy of incident photon will be divided into energy of scattered photon = *hν/* and energy of electron = *K.E*.



**Fig. 1. Compton Effect.**

For the conservation of energy, we can write

We know the momentum of a massless particle is related to its energy by

Since the energy of a photon is *hν,* its momentum is

Momentum is a vector quantity that incorporates direction as well as magnitude and in a collision momentum must be conserved in each of two mutually perpendicular directions. The directions we choose here are that of the original photon and one perpendicular to it in the plane containing the electron and the scattered photon (fig.1.).

The initial photon momentum is *hν/c* and the scattered photon momentum is *hν//c*, and the initial and final electron momenta are respectively *0* and *p*.

In the original photon direction

and perpendicular to this direction

The angle *θ* is that between the directions of the initial and scattered photons and, is that between the directions of initial photon and recoil electron.

Multiplying equations (4) and (5) by *c* we can write

Squaring equations (6) and (7) and adding, we get

We know the total energy

Again for all particles

Equating the above two equations we get

Squaring above equation we get

Since

We have

From equations (8) and (9) we can write

Dividing equation (10) by we get

Since

We can write

Equation (11) is the expression for the change in wavelength when an incident radiation of wavelength  is scattered by a free electron by an angle.

The quantity is called Compton wavelength of scattering particle.

Then equation (11) becomes

***Special Cases:***

1. When , then

Thus when, there is no change in wavelength.

1. When , then

Thus when, the change in wavelength is equal to Compton wavelength.

1. When , then

Thus when, the change in wavelength is maximum and equal to.

***Mathematical Problems***

***Problem-1:*** *A monochromatic X-ray of wavelength 0.124 A0 are scattered by a carbon block. Find the wavelength of X-rays scattered through an angle of 1800.*

***Solution:*** We know,

Here,

***Problem-2:*** *X-ray of = 1 A0 are scattered from a graphite block and the scattered radiation is viewed at 900 to the incident beam. What is Compton shift? Calculate the kinetic energy imparted to the recoiling electron.*

***Solution:*** We know,

Here,

The kinetic energy of the recoiling electron

***Problem-3:*** *X-rays of wavelength 10.0 pm are scattered from a target*

1. *Find the wave length of the X-ray scattered through 450*
2. *Find the maximum wavelength present in the scattered X-rays.*
3. *Find the maximum Kinetic energy of recoil electrons.*

***Solution:*** We know,

Here,

Maximum wavelength of the scattered X-rays

The maximum kinetic energy of the recoiling electron

**DE-BROGLIE MATTER WAVE**

***Wave Particle Dualism***

The concept of wave nature of matter arose from dual character radiation which sometimes behaves as a wave and at other times as a particle. For example, radiation considered as a wave in propagation experiments like interference, diffraction and polarisation. These experiments firmly establish the wave nature of radiation.

On the other hand the phenomenon like photo-electric effect and Compton Effect shows that radiation behaves as a particle- the photon. Thus radiation has a dual characteristic. However, radiation does not exhibit its wave and particle aspect simultaneously. Radiation thus, behaves as a wave transmission and as a particle when it interacts with matter.

In accordance with the theory of relativity Einstein had proved the equivalence of mass and energy i.e. E=mc2.

Similarly, in accordance with the quantum theory of radiation proposed by Max Planck a radiation frequency consist of quanta or photon each of energy h. These and other theoretical considerations lead de-Broglie to prove in 1924 that matter also had a dual (particle as well as wave –like) character. According to him electronns as well as protons which ordinarily behave like particles, under certain conditions, behave like a train of waves. The wavelength of these waves depends upon the momentum which in turn depends upon the mass and velocity of the particles.

The waves associated with a moving particle are called matter waves.

***de-Broglie wave***

In 1924 Luis de-Broglie proposed that *matter possesses wave as well as particle properties*. This wave is called de-Broglie wave.

A photon of light of frequency** has the momentum

The wavelength of a photon is therefore specified by the relation

This wavelength is called de-Broglie wavelength.

de-Broglie suggested that equation (1) is completely general one that applies to material particle as well as photons.

The momentum of a particle of mass *m* and velocity *v* is given by

and its de-Broglie wavelength is accordingly

***Thus, the greater the particle’s momentum, the shorter its wavelength***.

We know,

Again, we know

This is the expression for the Broglie wavelength in terms of energy.

***Wave velocity and group velocity***

The equation of plane progressive wave is given by

Here is called the angular frequency of the wave and *k* is called the wave vector.

Here is called the phase/wave velocity. In other words, is the velocity with which a plane progressive wavefront travels forward. It has a constant phase

Differentiating this equation with respect to t,

The **phase/wave velocity** of a [wave](http://en.wikipedia.org/wiki/Wave) is the rate at which the [phase](http://en.wikipedia.org/wiki/Phase_(waves)) of the wave [propagates in space](http://en.wikipedia.org/wiki/Wave_propagation). This is the [velocity](http://en.wikipedia.org/wiki/Velocity) at which the phase of any one [frequency](http://en.wikipedia.org/wiki/Frequency) component of the wave travels.

Or, equivalently, in terms of the wave's [angular frequency](http://en.wikipedia.org/wiki/Angular_frequency) *ω*, which specifies the number of [oscillations](http://en.wikipedia.org/wiki/Oscillation) per unit of time, and [wave number](http://en.wikipedia.org/wiki/Wavenumber) *k*, which specifies the number of oscillations per unit of space, by

The **group velocity** of a [wave](http://en.wikipedia.org/wiki/Wave) is the [velocity](http://en.wikipedia.org/wiki/Velocity) with which the overall shape of the waves' amplitudes-known as the modulation or [envelope](http://en.wikipedia.org/wiki/Envelope_(waves)) of the wave-propagates through space.

The group velocity vg is defined by the equation:

where ω is the wave's [angular frequency](http://en.wikipedia.org/wiki/Angular_frequency) (usually expressed in [radians per second](http://en.wikipedia.org/wiki/Radians_per_second)), and k is the [angular wave number](http://en.wikipedia.org/wiki/Angular_wavenumber) (usually expressed in radians per meter).

***Relation between wave and group velocity***

Let us consider that the wave group arises from the combination of two waves that have the same amplitude *A* but differ an amount *d* in angular frequency and an amount *dk* in wave number. If *y1* and *y2* are the displacement of the two waves respectively, then

The resultant displacement at any position *x* at any time *t* is given by

Since *d* and *dk* are small compared with ** and *k*

So that equation (3) can be written as

The wave velocity of a [wave](http://en.wikipedia.org/wiki/Wave) is

The group velocity of a [wave](http://en.wikipedia.org/wiki/Wave) is

The angular frequency and wave number of the de-Broglie wave group associated with a moving body of mass *m* moving with the velocity *v* are

The group velocity of de-Broglie wave group associated with a moving body is

Differentiating equation (5) with respect to *v* we get

Again, differentiating equation (6) with respect to *v* we get

From equation (7) we get the group velocity as

Thus, the de-Broglie wave group associated with a moving body travels with the same velocity as the body.

We can also say that *the group velocity is equal to the wave velocity*.

***Mathematical Problems***

***Problem-1:*** *If the de-Broglie wavelength of an electron is equal to . Calculate its kinetic energy.*

***Solution:*** We know

Here,